Appendix G: Fuel Burn Calculation[1]

This appendix describes more fully the fuel burn assumptions and methodologies used in this study. To calculate aircraft fuel burned, the following factors were considered: aircraft performance, aircraft weight, and flight trajectory. In many instances, aircraft performance data is not widely available from industry; therefore, alternative assumptions and methodologies must be considered and applied to calculate the fuel burned for the remaining aircraft that operate in the NAS.

Table G-1 is a list of all aircraft models for which detailed performance data was available for analysis. The aircraft performance data was derived from the FAA LINKMOD model. The data and its relative contribution to the total fuel consumed in the NAS were analyzed.

Table G-1. Aircraft Models for Which Detailed Performance Data was Available

Aircraft Model	Description
A300	Airbus 300
A310	Airbus 310
A320	Airbus 320
A330	Airbus 330
A340	Airbus 340
B727-100	Boeing 727-100
B727-200	Boeing 727-200
B737-200	Boeing 737-200
B73F	Boeing 737-400
B73S	Boeing 737-300
B73V	Boeing 737-500
B747-100	Boeing 747-100
B747-200	Boeing 747-200
B747F	Boeing 747-400
B757-200	Boeing 757-200
B767-200	Boeing 767-200
DC10-10	Douglas DC10-10
DC10-30	Douglas DC10-30
DC8-63	Douglas DC8
DC9-30	Douglas DC9-30
DC9-50	Douglas DC9-50
L1011	Lockheed L1011
MD11	McDonnell Douglas MD11
MD80	McDonnell Douglas MD80

An analysis of NASA CR-4700 indicated that the aircraft found in Table G-1 contributed to 87% of all fuel consumed globally. The remaining aircraft, for which fuel burn models was not

[1] This appendix was developed by Stephane Mondoloni (CSSI, Inc.) and Diana Liang (FAA/ASD-400).

available, affected only the remaining 13% of the fuel burn. It was therefore concluded that a fuel burn approximation for any aircraft not included in Table G-1 would have only a slight impact on results of the analysis.

As a secondary check on the relative contribution to total fuel burn, the total fuel consumption was computed on a day of traffic using actual flown traffic data and using the method described below. Similarly, the results indicate that 89% of all fuel consumed was attributable to those aircraft for which we had performance data.

Force balance equation

A force balance equation was used to calculate fuel burned for all aircraft listed in Table G-1. Once the trajectory and the model number were obtained for a flight, a numerical integration of the fuel weight was performed from the arrival to the departure point. This proceeded as a final value problem using an ordinary differential equation (ODE) describing the weight (W) summarized below. Note that it was assumed that the climb angle was small enough for the lift to be approximately equal to the weight.

$$C_L = \frac{W}{\frac{1}{2}\rho V^2 S}$$

$$Drag = C_D(C_L, M)\tfrac{1}{2}\rho V^2 S$$

$$T = Drag + W\sin(\gamma) - \tfrac{W}{g}\tfrac{dU}{dt}$$

$$\tfrac{dW}{dt} = -FF(T, h, M)$$

C_L - lift co-efficient	C_D - Drag co-efficient
M - Mach number	ρ- density
S - reference area	V - airspeed
W - weight	FF - fuel flow
T = thrust	t = time
h = altitude	γ - climb angle

Once the initial weight was found, the total fuel consumed for this flight was simply the initial weight minus the final weight.

Aircraft without performance data

For the remaining aircraft where detailed performance data was not available, the equation above reduces to the following:

$$\tfrac{dW}{dt} = W\left(k_1 + k_2\sin(\gamma) + k_3\tfrac{dU}{dt}\right)$$

The k's are constants to be determined through ordinary least squares (OLS) regression on the fuel flow obtained using the method described previously. A lower limit was imposed on the fuel flow to ensure against negative burn rates when aircraft are descending or decelerating rapidly.

In order to determine if the curve fitting approach was approximately valid for different types of general aviation aircraft, we obtained the fuel consumption (in gallons per hour) for different types of aircraft from the Aviation and Aerospace Almanac (1997). From the above equation, an average weight is implied by the average fuel consumption. Table G-2 shows that the implied weights are indeed typical for the aircraft listed.

Table G-2. Implied Weights given Fuel Consumption and Typical Aircraft

Type	Consumption GPH	Implied Weight LBS	Example Aircraft
Piston 1-3 seats	9.4	1178	Cessna 150 (985-1600lbs)
Piston 1-6 seats	26.6	3333	Piper PA-30 (2210-3600lbs)
Prop 1-12 seats	84.8	10626	Beech King Air (8500-14000lbs)
Jet 2 engines	263.2	32982	Dassault Falcon 2000 (19980-35000lbs)